

Surface Drawing with Sweeping using MLAB

A surface Q in 3-dimensional space may be represented in two parameter form as a function $Q(s, t) = (x(s, t), y(s, t), z(s, t))$. A special case of this is that the surface is the surface of a function. Then the parameter form of the surface is just $(x, y, z(x, y))$. In this paper, we will discuss a special case of surfaces in 3 dimensions. *i.e.* tube-like-surfaces that are generated by sweeping a space curve along another central space curve.

Given a space curve $r(t) = (x(t), y(t), z(t))$, at each point, there are three directions associated with it, the *tangent*, *normal* and *bi-normal* directions. The unit *tangent* vector is denoted by T , *i.e.* $T(t) = r'(t)/\|r'(t)\|$, the unit *normal* vector is denoted by N , *i.e.* $N(t) = T'(t)/\|T'(t)\|$, the unit *bi-normal* vector is denoted by B , *i.e.* $B(t) = T(t) \times N(t)$ (cross product). With $r(t), T(t), N(t)$ and $B(t)$, a tube-like surface can be expressed as follows:

$$Q(s, t) = r(t) + d \cdot (\cos(s)N(t) - \sin(s)B(t))$$

where d is a parameter corresponding to the radius of the rotation (In general d can be a function of t). For fixed t , when s runs from 0 to 2π , we have a circle around the point $r(t)$ in the N, B plane. As we change t , this circle moves along the space curve r , and we will generate a tube-like surface along r .

A simple example of this is a torus, where r is a circle. $r(t) = (\cos(t), \sin(t), 0)$. In this case, $T(t) = (-\sin(t), \cos(t), 0)$, $N(t) = (-\cos(t), -\sin(t), 0)$ and $B(t) = (0, 0, 1)$. Thus, the parametric form of the torus is:

$$\begin{aligned}x(s, t) &= \cos(t) - d \cdot \cos(s)\cos(t) \\y(s, t) &= \sin(t) - d \cdot \cos(s)\sin(t) \\z(s, t) &= d\end{aligned}$$

We can easily construct and draw such a torus in *MLAB* by merely computing the x, y and z functions on a regular grid of (s, t) -points. Using the functions x, y and z , this can be done as follows:

```
grid = cross((0:(2*pi)!30, 0:(2*pi)!30)
d = 0.5
m = (x on grid) &' (y on grid) &' (z on grid)
draw m lt hide
```

Note that the *MLAB* draw statement is able to correctly discover the underlying grid structure in the matrix *m* automatically!

We can also build a general *do-file* which draws a tube-like-surface given a central curve and a radius function. Here is such an *MLAB do-file*.

```
"tube.do"
"generate a tube of radius d(t) around the curve: r(t) = (x(t),y(t),z(t))"
"The central space curve component functions x(t), y(t) and z(t) must be"
"predefined. The radius function d(t) must also be predefined, and"
"finally the (s,t) grid matrix called grid must also be predefined."

"Construct unit tangent vector T(t)"
"nd(t) is the norm of the derivative"
fct nd(t) = ((x't(t))^2+(y't(t))^2+(z't(t))^2)^0.5
"a(t), b(t) and c(t) are the component of T(t)"
fct a(t) = x't(t)/nd(t)
fct b(t) = y't(t)/nd(t)
fct c(t) = z't(t)/nd(t)

"Construct unit normal vector N(t)"
"ndut(t): norm of the derivative of the unit tangent vector"
fct ndut(t) = ((a't(t))^2+(b't(t))^2+(c't(t))^2)^0.5
"an(t), bn(t) and cn(t) are the component of N(t)"
fct an(t) = a't(t)/ndut(t)
fct bn(t) = b't(t)/ndut(t)
fct cn(t) = c't(t)/ndut(t)

"Construct the unit binormal vector B(t)"
"ab(t), bb(t) and cb(t) are the component of B(t)"
fct ab(t) = b(t)*cn(t)-c(t)*bn(t)
fct bb(t) = c(t)*an(t)-a(t)*cn(t)
fct cb(t) = a(t)*bn(t)-b(t)*an(t)

"Now, construct the space tube"
"m(s,t) = r(t) + d*(cos(s)*N(t) + sin(s)*B(t))"
"d is the radius of the tube along the central curve"
"mx(s,t), my(s,t), mz(s,t) is the component of m(s,t)"
fct mx(s,t) = x(t) + d(t)*(cos(s)*an(t)+sin(s)*ab(t))
fct my(s,t) = y(t) + d(t)*(cos(s)*bn(t)+sin(s)*bb(t))
fct mz(s,t) = z(t) + d(t)*(cos(s)*cn(t)+sin(s)*cb(t))

"now generate the surface"
mm = (mx on grid) &' (my on grid) &' (mz on grid)
```

```

draw mm, linetype net
view
"end of tube.do"

```

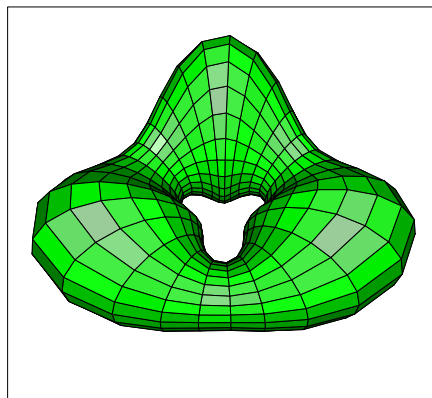
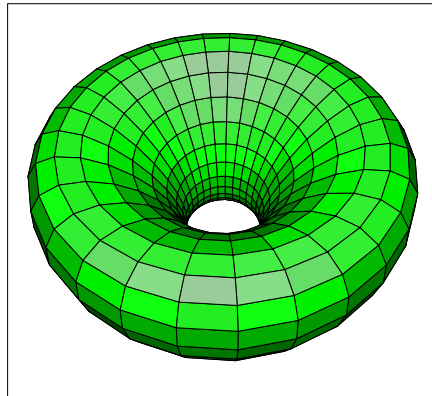
Here is an example showing the use of tube.do to draw a torus.

```

"the central line curve is (cos t, sin t, 0)"
"define the center space curve r(t) = (x(t), y(t), z(t))"
fct x(t) = cos(t)
fct y(t) = sin(t)
fct z(t) = 0
grid = cross((0:(2*pi)!30), (0:(2*pi)!30))
fct d(t) = 1
do tube.do

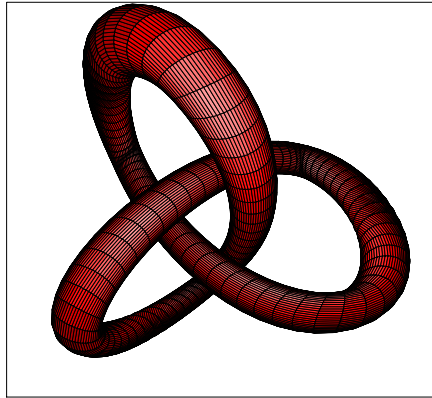
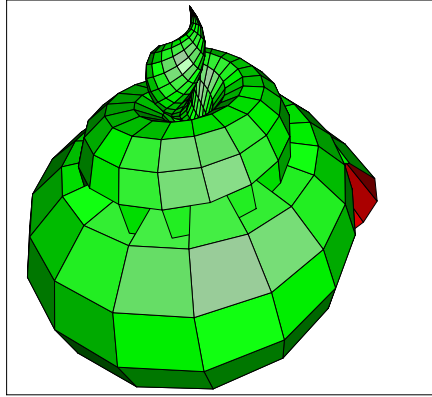
```

Here is some output from the above *do-file* tube.do.



Central curve $r(t) = (\cos(t), \sin(t), 0)$. Left: a fixed radius. Right: a variable radius.

One can generate more interesting surfaces with other central curves. Here are two examples with two different central curves.



Left: Descending central curve and ascending radius. Right: self crossing central curve and constant radius.

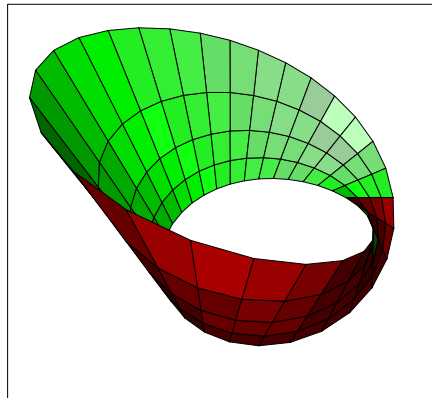
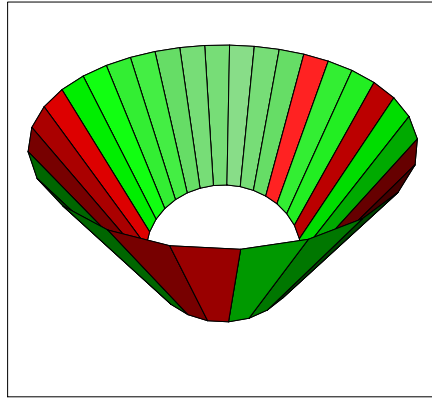
Generally speaking, our sweeping curve does not need to be a circle, it can be any 2-dimensional curve $c(s, t) = (p(s, t), q(s, t))$. Thus, instead of having

$$Q(s, t) = r(t) + d \cdot (\cos(s)N(t) - \sin(s)B(t))$$

as the sweeping surface, we have:

$$Q(s, t) = r(t) + p(s, t) \cdot N(t) - q(s, t) \cdot B(t)$$

In particular, when the curve c is a straight line segment, the sweeping surface becomes a band. Here are two such bands: one with a fixed line segment, and one with a rotating line segment which is the famous *Mobius* band.



Left: Regular band with fixed radius and direction. Right: Mobius band.

One can easily see that if we adjust the radius, direction, number of points on each circle etc, we can generate many interesting surfaces.