File: /General/MLAB-Text/Papers/dimer/dimer.tex

Chemical Kinetics Modeling

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The MLAB advanced mathematical and statistical modeling system is an unparalleled tool for mathematical modeling; take a look at the following example. Only MLAB can solve a kinetics-modeling problem like this so easily! (It's just as easy to solve enzyme kinetics, multiple site binding equilibrium, or any other of a wide variety of problems.)

Below we show an interesting example of MLAB modeling for dimer kinetics. Suppose we have two substances, A and B which bind to form a complex C, and the substance C, in turn, binds with itself to form a dimer D. We thus have:

$$A + B \underset{k_2}{\overset{k_1}{\rightleftharpoons}} C, \qquad C + C \underset{k_4}{\overset{k_3}{\rightleftharpoons}} D.$$

Suppose further we mix 2 mmoles of A and 3 mmoles of B and measure the concentration in mmoles of both C and D at ten equally-spaced times between 7 and 70 seconds. From this data we wish to estimate the association and dissociation constants k_1, k_2, k_3, and k_4. We may proceed in MLAB as follows.

First we read in the data consisting of values of $c(t)$ and $d(t)$ given at the common times $7 : 70!10$. Although common times are used here, this is *not* required.

```
*data = (7:70!10) &' read(ddata,10,2)
*type data

     time    c       d

  1: 7       1.065   0.0058
  2: 14      1.383   0.2203
  3: 21      0.9793  0.4019
```

```
  4: 28      1.107   0.3638
  5: 35      0.7289  0.456
  6: 42      0.7236  0.5014
  7: 49      0.4674  0.715
  8: 56      0.6031  0.4723
  9: 63      0.6149  0.7219
 10: 70      0.3369  0.7294

*cdata = data col 1:2
*ddata = data col (1,3)
```

Now we define our kinetic model so that $c(t)$ is the concentration of c in mmoles at time t and $d(t)$ is the concentration of d in mmoles at time t.

```
* fct c't(t)=k1*(a0-c-2*d)*(b0-c-2*d)-k2*c-2*d't(t)
* fct d't(t)=k3*c*c-k4*d
* initial c(0)=0
* initial d(0)=0
* a0=2;b0=3
```

Now we guess the values of k_1, k_2, k_3, and k_4. We may use the results of equilibrium studies, analyzed by MLAB, to know values for the ratios k_1/k_2 and k_3/k_4.

```
* k1=.02;k2=.002; k3=.02;k4=.002
* constraints q={k1>0,k2>0,k3>0,k4>0}
```

Now we may curve-fit the two ode-system-defined functions, c and d, to estimate k_1, k_2, k_3, and k_4.

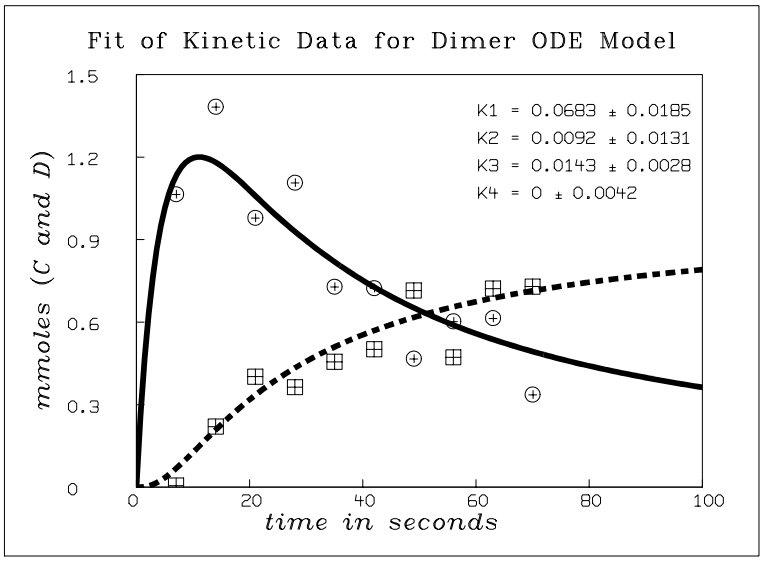
```
* fit(k1,k2,k3,k4), c to cdata, d to ddata, constraints q
final parameter values
      value              error                dependency    parameter
 0.06830069826       0.01853684638          0.5090866122   K1
 0.009215472845      0.01307266967          0.5011226583   K2
 0.01429036692       0.002818099621         0.7828534111   K3
 1.616224063e-19     0.004158477431         0.7620523838   K4
6 iterations
CONVERGED
best weighted sum of squares = 2.222001e-01
weighted root mean square error = 1.178453e-01
```

```
weighted deviation fraction = 1.203378e-01
lagrange multiplier[4] = -3.928493584
```

Now we may draw the results of the curve-fit.

```
* m=integrate(c't,d't,0:100!140)
* draw m col (1,2) color red
* draw m col (1,4) color green lt dashed
* draw cdata pt circle lt none color red
* draw ddata pt circle lt none color green
* bottom title "time in seconds"
* left title "mmoles (C and D)"
*
* oformat = nformat; nformat ="%4.4lf"
* v=strval(stdest[1]); s=strval(k1)+" '25TF'R "+substr(v,7:strlen(v))
* title s at (.6,.8) ffract size .015
* v=strval(stdest[2]); s=strval(k2)+" '25TF'R "+substr(v,7:strlen(v))
* title s at (.6,.75) ffract size .015
* v=strval(stdest[3]); s=strval(k3)+" '25TF'R "+substr(v,7:strlen(v))
* title s at (.6,.7) ffract size .015
* v=strval(stdest[4]); s=strval(k4)+" '25TF'R "+substr(v,7:strlen(v))
* title s at (.6,.65) ffract size .015
* nformat=oformat

* view
```


Fit of Kinetic Data for Dimer ODE Model
K1 = 0.0683 ± 0.0185
K2 = 0.0092 ± 0.0131
K3 = 0.0143 ± 0.0028
K4 = 0 ± 0.0042
mmoles (C and D)
time in seconds
0
0.3
0.6
0.9
1.2
1.5
0
20
40
60
80
100