

Local Maxima and Minima in Time-Series

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MLAB has many unusual operators not commonly found in most mathematical and statistical software packages. Two such operators are the threshold local minima and threshold local maxima functions.

The threshold local minima function, $LMIN(V[,t])$, computes a matrix consisting of those rows of the input matrix V for which the entries in last column of V hold the local minima with threshold t . The optional threshold value t defaults to zero if it is omitted.

Given a list of values v_1, v_2, \dots, v_n , the value v_i is a threshold- t local minimum if there exists a neighborhood $v_j, v_{j+1}, \dots, v_i, v_{i+1}, \dots, v_h$ of v_i with $j < i < h$ such that

- (1) $v_i = \min(v_j, \dots, v_h)$
- (2) $v_i < v_k$ for $k = j, \dots, i - 1$
- (3) $v_k - v_i \leq t$ for $k = j + 1, \dots, h - 1$
- (4) $v_j - v_i > t$ and $v_h - v_i > t$

The threshold local maxima function, $LMAX(V[,t])$, computes a matrix consisting of those rows of the input matrix V for which the entries in last column of V hold the local maxima with threshold t . The optional threshold value t defaults to zero if it is omitted.

Given a list of values v_1, v_2, \dots, v_n , the value v_i is a threshold- t local maximum if there exists a neighborhood $v_j, v_{j+1}, \dots, v_i, v_{i+1}, \dots, v_h$ of v_i with $j < i < h$ such that

- (1) $v_i = \max(v_j, \dots, v_h)$
- (2) $v_i > v_k$ for $k = j, \dots, i - 1$
- (3) $v_i - v_k \leq t$ for $k = j + 1, \dots, h - 1$
- (4) $v_i - v_j > t$ and $v_i - v_h > t$

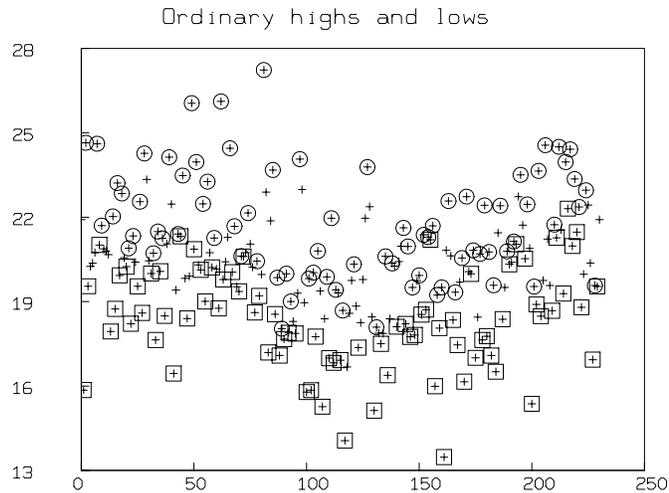
Suppose we wish to determine the successive points at which a stock price hits a new high or a new low. this can be done by finding all the local maxima and local minima with the $LMAX$ and $LMIN$ operators.

Suppose however, we want to define a new high or new low as a price which is more extreme than its ranges of neighboring prices by a certain percentage, say 5%. This isn't quite a precise concept, because exactly what are we assessing 5% of? Moreover, the LMAX and LMIN operators deal with absolute, not relative thresholds. We may resolve these problems by normalizing our prices relative to their moving average, *i.e.* by scaling a price $p(t)$ by the value $1/m(t)$ where $m(t)$ is a suitable moving average price at time t . We may then apply LMAX and LMIN to this scaled time-series with the desired threshold value.

Here is an example of this device for a particular time-series of 230 prices at successive time-points.

```
p=read(stock,230,2) /* read in data from file stock.dat */

draw p pt crosspt psize .01 lt none
hi=lmax(p)
lo=lmin(p)
draw hi pt circle lt none color red
draw lo pt square lt none color green
top title "Ordinary highs and lows"
view
```



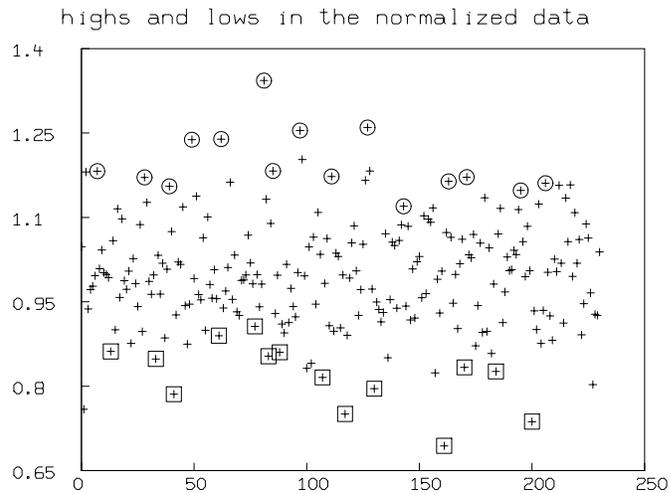
del w

```

wm=(0:1!15)&(1^^15)&(1:0!15)
v=mmean(p col 2,45,0,4,wm)
m=(p col 1)&'v
s=(p col 1)&'((p col 2)/'v)
draw s pt crosspt ptsize .01 lt none

hi=lmax(s,.3)
lo=lmin(s,.3)
draw hi pt circle lt none color red
draw lo pt square lt none color green
top title "highs and lows in the normalized data"
view

```



```

del w

draw p pt crosspt ptsize .01 lt none
draw m color yellow
hi col 2 = (hi col 2)*'(v row (hi col 1))
lo col 2 = (lo col 2)*'(v row (lo col 1))
draw hi pt circle lt none color red
draw lo pt square lt none color green
top title "adjusted highs and lows"

```

view

