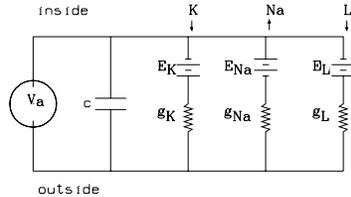


## An MLAB Example: The Hodgkin-Huxley Nerve Axon Model

The Hodgkin-Huxley model of an axon membrane is a mathematical description of the potential difference or voltage across the membrane which changes as a function of time in response to various perturbations of potential established with an associated applied current.

This description is in the form of an analogous circuit which in turn can be characterized by a set of non-linear differential equations. These equations contain a description of the feedback system which changes the conductances (or equivalently, the permeabilities) of the membrane as a function of time and potential difference. The circuit merely contains the three main ionic current circuits in parallel together with the observed membrane capacitance.

We may apply a time-varying current,  $I_a$ , across the membrane as a stimulus. By applying whatever time-varying voltage,  $V_a$ , across the membrane is needed to achieve the current  $I_a(t)$  at time  $t$ .



Thus, in general, we have  $I_a = I_K + I_{Na} + I_L + I_c$ , where  $I_c$  is the current across the membrane due to the capacitance  $c$ . Now  $I_c = c(dE_m/dt)$ , where  $E_m$  is the potential across the membrane measured in volts and  $c \approx 1 \mu\text{farad}/\text{cm}^2$ . Thus, we obtain

$$c \frac{dE_m}{dt} = I_a - g_K(E_m - E_K) - g_{Na}(E_m - E_{Na}) - g_L(E_m - E_L).$$

The Hodgkin-Huxley model assumes that the emf's  $E_K$ ,  $E_{Na}$ , and  $E_L$ , and the conductance  $g_L$ , are constant, but that the conductances  $g_K$  and  $g_{Na}$  are not constant, but rather are functions of the membrane potential difference history.

A change in  $g_K$  corresponds to a change in the permeability of the membrane for potassium and similarly for  $g_{Na}$ . The equations below adequately serve the purpose of defining  $g_K$  and  $g_{Na}$  as functions of time and membrane potential difference.

The Hodgkin-Huxley equations are an example of an ODE model that can be studied using the MLAB computer program. MLAB is an advanced mathematical and statistical modeling system applicable to a wide variety of data analysis and experimental mathematical problems in the sciences.

Below we present the complete Hodgkin-Huxley model as MLAB statements.

```
"file: hhf.do";
el=-50; ena=55; ek=-72;
gkbar=36; gnabar=120; gl=.3179676;

function phi(temp)=3^((temp-6.3)/10);
function f(x)=if abs(x)<.00002 then 1 else x/(exp(x)-1);
function bm(e)=4*exp((-e-60)/18);
function bh(e)=1/(1+exp(-3-.1*e));
function bn(e)=exp((-e-60)/80)/8;
function am(e)=f((-35-e)/10);
function ah(e)=.07*exp((-e-60)/20);
function an(e)=.1*f((-50-e)/10);
function minf(e)=am(e)/(am+bm(e));
function hinf(e)=ah(e)/(ah+bh(e));
function ninf(e)=an(e)/(an+bn(e));

k=25; s=.2;
function ia(t)=i0+(i1-i0)*(if t<s then 1-exp(-k*t) else \
(1-exp(-k*s))*exp(-k*(t-s)));
```

```

function ionic(e,m,h,n)=gnabar*m^3*h*(e-ena)+ \
    gkbar*n^4*(e-ek)+gl*(e-el);
function em't(t)=ia(t)-ionic(em,m,h,n);
function m't(t)=phitemp*(am(em)*(1-m)-bm(em)*m);
function h't(t)=phitemp*(ah(em)*(1-h)-bh(em)*h);
function n't(t)=phitemp*(an(em)*(1-n)-bn(em)*n);
function iss(e)=ionic(e,minf(e),hinf(e),ninf(e));
function ess(i)=root(e,-246,830,iss(e)-i);
"for -62<i<32751, ess(i) lies between -246 and 830";

temp=6.3; phitemp=phi(temp);
v0=-60; i0=0; ess0=ess(i0); vi=0; i1=0;

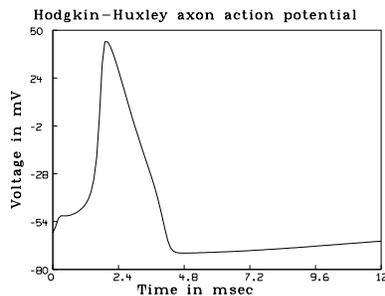
initial em(0)=ess0+vi; initial m(0)=minf(ess0);
initial h(0)=hinf(ess0); initial n(0)=ninf(ess0);

```

The user can establish any desired boundary conditions and then solve the Hodgkin-Huxley equations in MLAB.

```
q=integrate(em't,m't,h't,n't,0:12);
```

One such resulting graph of  $E_m$ , with added titles, is given below as produced in MLAB.



Many familiar observations can be simulated and the model's response can be investigated by solving the Hodgkin-Huxley equations with various initial conditions. See "Computer analyses of the excitable membrane" by T. Hironaka and S. Morimoto in *Computers and Biomedical Research*, Vol. 13, pp. 36:51, 1980, for a number of interesting numerical experiments which can be performed.

One of the main agreements of the Hodgkin-Huxley model with experiment is the suitability of the function  $E_m(t)$  as a solution to the cable equation which involves the velocity of an action potential wave propagating along an axon.

Although not exhibited here, MLAB has powerful facilities for fitting models to data, as well as a large set of functions in mathematics and statistics that are of use in biomedical research. More information can be obtained from Civilized Software, Inc., 12109 Heritage Park Circle, Silver Spring MD 20906 USA, Tel.: (301)-962-3711, URL: [www.civilized.com](http://www.civilized.com)