File: /General/MLAB-Text/Papers/sarea/sarea.tex

Computing Surface Area Using MLAB

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Given a function $f(x,y)$ and a rectangular region $A = [xmin, xmax] \times [ymin, ymax]$, we wish to compute the surface area of $f(x,y)$ over A. A standard formula for computing the surface area of a function $f(x,y)$ is:

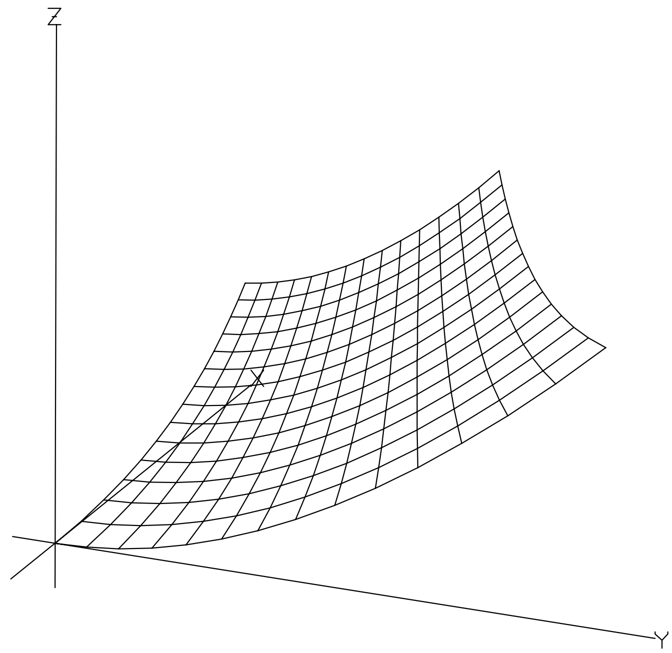
$$\begin{aligned} Area &= \int_A \sqrt{1 + f_x^2 + f_y^2}\, dx\, dy \\ &= \int_{xmin}^{xmax} \int_{ymin}^{ymax} \sqrt{1 + f_x^2 + f_y^2}\, dx\, dy \end{aligned}$$

Where, f_x and f_y denotes the partial derivatives of f with respect to x and y. Thus, to compute the surface area of $f(x,y)$ over A, we just need to write the above formula in *MLAB*. Here is an example which shows the symbolic differentiation and numerical integration abilitiies of *MLAB*.

```
fct f(x,y) = x^2 + y^2  /* surface function */
xmin = 0; xmax = 1; ymin = 0; ymax = 1 /* region boundaries */

fct g(x,y) = sqrt(1 + (f'x(x,y))^2 + (f'y(x,y))^2)
fct q(x) = integral(y, ymin,ymax, g(x,y))
fct a() = integral(x, xmin, xmax, q(x))

type a() /* function value */

    = 1.86156384
```

Here is the graph of the surface whose area is computed.

```
m = points(f, cross((xmin:xmax!15), (ymin:ymax!15)))
draw m lt net

view
```



Note that the above area function `a()` can also be written in one step without using `q(x)`. *i.e.*

```
fct a() = integral(x, xmin, xmax, integral(y, ymin,ymax, g(x,y)))
```

One can also do non-rectangular region surface computation by writing out the lower and upper bound of the double integral in explicit functional format.